Lab 7: Fabry-Perot Interferometer

1 Introduction

The Fabry-Perot interferometer is fundamentally different from a Michelson because it exploits multiple reflections of a single beam between two parallel mirrors, rather than two beams that travel different paths (see Fig. 1). It consists of two parallel mirrors, one of which may be movable, with light perhaps incident at a small angle θ. Some light is transmitted by the first mirror, reflected back by the second one, and then undergoes additional reflections. It is also an instrument of extraordinary sensitivity, and its operation depends on the relative phase of the light beams that have undergone different numbers of reflections between the mirrors.

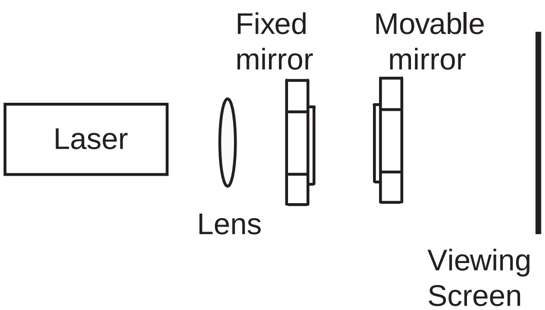

Figure 1: Schematic view of the Fabry-Perot Interferometer.

2 How it Works

The accumulated phase difference δ on each round trip between two mirrors of separation L is given by $\delta = (2\pi)(2L\cos\theta/\lambda)$.

Question 1: This result is **NOT** simple - show geometrically how to find it!.

We sum the various contributions transmitted through the second mirror from the multiple reflections to the electric field $\mathcal{E}_T$, taking this phase difference into account. We suppose the mirrors have reflectance r and transmittance t, and we find the sum

$$\mathcal{E}_T = \mathcal{E}_0 t^2 + \mathcal{E}_0 t^2 r^2 e^{i\delta} + \mathcal{E}_0 t^2 r^4 2 e^{2i\delta} + ... \tag{1}$$

where $\mathcal{E}_0$ is the electric field of the incident light. This is a geometric series whose sum is

$$\mathcal{E}_T = \frac{\mathcal{E}_0 t^2}{1 - r^2 e^{i\delta}} \tag{2}$$

and so the transmitted intensity, proportional to $|\mathcal{E}|^2$ is

$$I_T = I_0 \frac{T^2}{|1 - R\, e^{i\Delta}|^2} \tag{3}$$

where $T \equiv tt^*$, $R \equiv rr^*$, and $\Delta \equiv \delta + \delta_r$. Note that $T + R = 1$. The reason for the r^*, etc. expressions is that there may be a phase shift δ_r upon reflection, so r may be complex. We can readily evaluate the denominator and find

$$I_T = I_0 \frac{T^2}{1 - R^2} \, \frac{1}{1 + F\sin^2(\Delta/2)} \tag{4}$$

where $F \equiv 4R/(1-R)^2$ is called the finesse of the interferometer. Clearly for the resonant case, $\sin\Delta = 0 \Rightarrow$ cavity length is an integer multiple of $\lambda/2$ and normal incidence, the transmission is 100%, independent of the value of r. For a graph of I_T see page 89 of Fowles.

Question 2: Derive Eq. 4.

This is a most amazing result. If we shine a beam of light on a 99% reflecting mirror, 1% goes through and the other 99% is reflected. Yet if we place a second mirror **behind** the first one, where there is only 1% of the light present, somehow 100% of the light appears.

Question 3: Can you explain this??? How can energy be conserved if the first mirror reflects 99% of the light and yet 100% is transmitted??

The ratio

$$\frac{I_{T,\mathrm{max}} - I_{T,\mathrm{min}}}{I_{T,\mathrm{min}}} = \frac{1/(1+0) - 1/(1+F)}{1/(1+F)} = \frac{1 - 1/(1+F)}{1/(1+F)} = 1 + F - 1 = F \tag{5}$$

tells us that the contrast between max and min is F. Since we can get F as high as 10^6 we can make really high contrast fringes. At the half-max intensity point where $\Delta = \Delta_c$, we find $1/2 = 1/(1 + F\sin^2(\Delta/2))$ so $\Delta_c = 2/\sqrt{F}$ since $\Delta_c \ll 1$ and we make the small angle approximation for $\sin\Delta_c$. Thus the full width at half max (FWHM) is $4/\sqrt{F}$.

3 Experiment

Clearly the way to scan this interferometer to see the variations of transmitted intensity is to move one of the mirrors with respect to the other. But it's very hard indeed to do this and keep them parallel, so instead we'll use a diverging light beam and observe fringes caused by the variation of θ.

The setup will be similar to that of the Michelson interferometer setup, but there will be no beam-splitter. Position the stationary mirror so that its face is parallel to the movable mirror and just a few millimeters or so away from it. Mount the viewing screen behind the movable mirror. Adjust the stationary mirror until you get just one spot on the screen. Now mount the 18 mm lens on a holder about 6 cm from the stationary mirror to produce a diverging beam and thus a continuum of values of θ. You should see clear fringes on the screen. Find the center of the fringe pattern. Project the fringes on a screen far away.

Measure the angular sizes of a series of fringes. The theoretical relationship between the order of the fringe, the wavelength of the light, the angular displacement of the fringe, and the distance between the mirrors can be found from the derivation above. Measure the FWHM of the fringes and compare with $4/\sqrt{F}$ (find F by measuring the reflection coefficient, or more easily, the transmission coefficient) of the mirrors. Discuss the sources of error in this method.