PHY 123 Lab #7 – Conservation of Angular Momentum

The purpose of this lab is to study the relationship among torque, moment of inertia, angular acceleration and the conservation of angular momentum.

Equipment

- LabQuest 2
- Vernier rotary motion sensor
- Vernier rotary motion accessory kit
- Vertical support rod
- Balance
- Metric ruler

Introduction

In Part I we will measure the angular acceleration α of a system under a known external torque taking in to account friction and will use out measurement to obtain the moment of inertia of the system. The equation which relates the net torque τ_{net} to the moment of inertia and the angular acceleration of a rotating object is

$$\tau_{net} = I\alpha \qquad (1)$$

We will measure the angular acceleration by measuring the angular velocity ω as a function of time t. For a constant angular acceleration the equation which gives the dependence of ω on t is

$$\omega = \omega_0 + \alpha t \qquad (2)$$

In Part II, conservation of angular momentum is investigated. The definition of angular momentum L is

$$L = I\omega \qquad (3)$$

We will test whether angular momentum is conserved before and after the collision, using the relationship

$$I\omega = I'\omega' \qquad (4)$$

The dashed quantities refer to the angular momentum and angular velocity in the final state (after collision), and the undashed quantities refer to same variables in the initial state (before collision). In order to increase the moment of inertia of the rotating object we will drop an aluminum disk onto the rotating platform, thus changing the moment of inertia, and measure the angular velocity ω before and after the drop.

The apparatus used for this lab is sketched in the figure below. The Vernier rotary motion sensor is a bidirectional motion sensor that measures angular motion precisely and plots angular displacement,

angular velocity and angular acceleration vs. time. This sensor is direction sensitive, meaning the signs of the vectors depend on the direction of the rotation of the sensor. For this experiment, we do not need to record the sign of each quantity, only the absolute value.

The motion sensor comes with a 3-step pulley (10mm, 29mm, 48mm diameter). A lightweight string can be wound around the pulleys; here we will only be using the largest one. When a small mass m is attached to the other end of the string, the tension T due to this small mass will create an external torque τ_{ext}. The system is **not** frictionless. A frictional torque, τ_{fr} , opposes the external torque resulting in a net torque, $\tau_{net} = \tau_{ext} - \tau_{fr}$. The net torque gives the rotating platform an angular acceleration α which is measured by the rate of increase of the angular velocity. The frictional torque τ_{fr} causes a frictional deceleration α_{fr} .

The moment of inertia I of the rotating platform can be calculated from

$$I = \frac{mr(g-r\alpha)}{|\alpha_{fr}|+\alpha} \quad (5)$$

Where g is the acceleration due to gravity and r is the radius of the 3-step pulley around which you wind the string (0.024 m if you wind the string around the largest diameter).

This equation can be derived by considering that the net torque is $\tau_{net} = \tau_{ext} - \tau_{fr} = I\alpha$. the frictional torque is $\tau_{fr} = I\alpha_{fr}$ and the applied torque is $\tau_{ext} = Tr = m(g-a)r$.

Procedure: Measurement of α_{fr}

1. Connect the Rotary Motion Sensor to your data-collection interface and set the time to 20 seconds (by clicking in "Time" on the Meter screen).
2. Give the aluminum disk attached to the pulley a push so that it starts rotating slowly and then click "Collect" on the LabQuest2.
3. Go to the "Graph" screen, take the velocity vs. time graph and fit a straight line to your data by selecting the data on your graph and going to "Analyze" and then "Curve Fit". Select "Linear" in the fit equation option, and then click OK.
4. Record the value of the slope, which represents the angular acceleration α_{fr}.
5. Repeat the measurement twice for a total of three measurements and consider the uncertainty in α_{fr} to be half the difference between your maximum and minimum value of α_{fr}.

Measurement of α

1. Wind the string around the biggest diameter of the 3-step pulley and attach the small mass (10 g) to the other end of the string. Make sure your string is long enough in order to collect enough data points when the weight of the mass m falls toward the floor.

2. Click Collect data from LabQuest2 and release the system. Click stop once the mass has reached the lowest height.
3. Go to the "Graph" screen, take the velocity vs. time graph and fit a straight line to your data.
4. Record the value of the slope, which represents the angular acceleration α.
5. Repeat the measurement twice for a total of three measurements and consider the uncertainty in α to be half the difference between your maximum and minimum value of α.

Calculating *I*

We have now measured everything we need to calculate the moment of inertia of the system using equation 5. The uncertainty in I will be mostly due to the uncertainties in your measured values of α and α_{fr}. The uncertainty in I can therefore be estimated using the formula

$$\frac{\Delta I}{I} = \sqrt{\left(\frac{\Delta\alpha}{\alpha}\right)^2 + \left(\frac{\Delta\alpha_{fr}}{\alpha_{fr}}\right)^2} \qquad (6)$$

Testing of Conservation of Angular Momentum for Aluminum disks collision

Now that we know what the initial moment of inertia of the system is, we will perform an experiment in which we change it by increasing its mass. This is done by dropping an additional disk on top of the rotating platform. This drop (if done carefully) does not cause an external torque which would modify the initial angular momentum.

We need to calculate the moment of inertia of the disk I_{disk} that is dropped from its mass M and its radius R, using the formula.

$$I_{disk} = \frac{1}{2}MR^2 \quad (7)$$

Measure the radius R of the disk with the ruler (approximately 24 mm) and use 0.5 mm for its absolute error. Obtain the value of M and calculate the value of ΔI_{disk} taking in to account only the error in R:

$$\frac{\Delta I_{disk}}{I_{disk}} = \left|2\right|\left(\frac{\Delta R}{R}\right) \qquad (8)$$

The final moment of inertia I' is equal to the sum of I (from previous section) and I_{disk}, the moment of inertia of the dropped disk. For the total moment of inertia I' of the combined object (platform + dropped disk) take simply the sum of the two moments of inertia, $I' = I + I_{disk}$.

Write your value for I' in your notebook. This value is only valid if the axis of rotation goes through both the center of the rotating table and the center of the dropped disk (When you calculated I_{disk} you used

an equation which was valid only if the axis of rotation went through its center). Find the uncertainty in I' by propagating the error of I and I_{disk}:

$$\Delta I' = \sqrt{\left(\Delta I\right)^2 + \left(\Delta I_{disk}\right)^2}$$

(9)

Remove the string from the 3-step pulley. Spin the rotating table. Press the "Collect" button, while the rotating table is spinning, drop the wide-hole aluminum disk.

Go to the "Graph" screen and you should get an angular velocity vs. time graph that looks roughly like this:
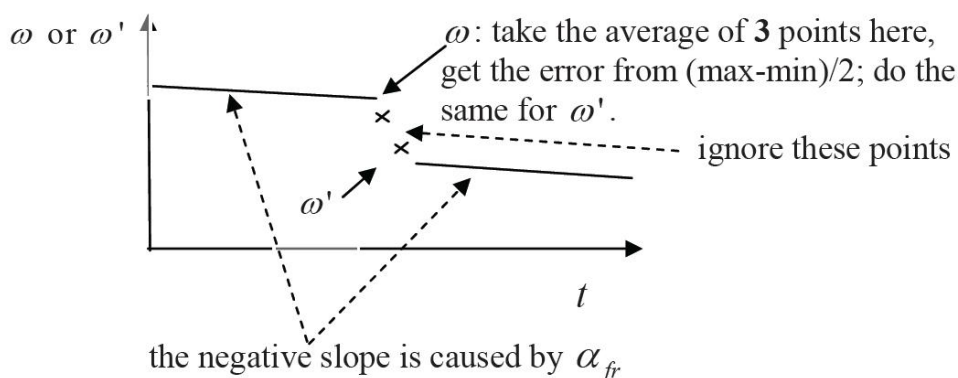
Note: The sign of the values for ω and ω' (and the slope) may change depending on the direction the disk is spinning in. In general, you will expect to have a graph with a jump in the values of ω due to the dropped disk.

The higher line set of points corresponds to the initial angular velocity ω and the lower line set of points corresponds to the final angular velocity ω'. From the table of values record the angular velocity for 3 points on each side of the jump ω→ω' in your notebook (values close to the jump).

Calculate the average value of ω and ω' and use half the difference between the maximum and minimum values of ω and ω' as an estimate of the error in these two quantities.

You are now ready to work out L and L' from equation (3). You should also calculate the error in L and L' taking in to account the uncertainties in the moments of inertia and the measured angular velocities. Once we have done this we can see if L and L' are the same within experimental uncertainty:

$$\frac{\Delta L}{L} = \sqrt{\left(\frac{\Delta I'}{I'}\right)^2 + \left(\frac{\Delta \omega}{\omega}\right)^2}$$

(10)

Answer the following questions:

1. Was angular momentum conserved in your experiment (within experimental error)? If not, what are possible reasons?
2. Assume that you will test the conservation of angular momentum one more time, but now, you will keep the string and the hanging mass in the system. In other words, you wind the string around the pulley, with the mass on the other side and then, you will release the system. Before the mass reaches its lowest point you drop the other aluminum disk and take the values of ω and ω' from the graph. Would you expect to observe conservation of angular momentum in this situation? Why or why not?